$(Q\bar{s})^{(*)}(\bar{Q}s)^{(*)}$ molecular states from QCD sum rules: a view on $Y(4140)$

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Masses for the $(Q\bar{s})^{(*)}(\bar{Q}s)^{(*)}$ ($Q = c$ or b) molecular states are systematically computed in the framework of QCD sum rules. Technically, contributions of the operators up to dimension six are included in operator product expansion (OPE). The numerical result 4.13 ± 0.10 GeV for $D_s^*\bar{D}_s^*$ agrees well with the mass $4143.0 \pm 2.9 \pm 1.2$ MeV for $Y(4140)$, which supports the $D_s^*\bar{D}_s^*$ molecular configuration for $Y(4140)$.

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I. INTRODUCTION

Recently, the CDF Collaboration has reported the observation of a narrow near-threshold structure in the $J/\psi\phi$ mass spectrum in $B^+ \to J/\psi\phi K^+$ decays [1], for which the mass is $4143.0 \pm 2.9 \pm 1.2$ MeV and the width is $11.7^{+8.3}_{-5.0} \pm 3.7$ MeV. This experimental observation has triggered great interest of many practitioners, and there have already appeared some theoretical interpretations for this new resonance, e.g. Refs. [2, 3, 4, 5, 6]. On the whole, $Y(4140)$ is apt to be deciphered as the molecular partner of the charmonium-like state $Y(3930)$ [7]. Undoubtedly, the quantitative description of $Y(4140)$'s properties such as mass is quite needed for well understanding its structure, but it is difficult to extract information on the hadronic spectrum from the rather simple Lagrangian of QCD. That's because low energy QCD involves a regime where it is futile to attempt perturbative calculations and one has to treat a genuinely strong field in nonperturbative methods. Whereas, one can resort to QCD sum rules [8] (for reviews see [9, 10, 11, 12] and references therein), which is a nonperturbative analytic formalism firmly entrenched in QCD. In fact, some authors [13, 14] have studied $Y(4140)$ via QCD sum rules soon after its observation, however, they arrived at different conclusions basing on the $D_s^*\bar{D}_s^*$ molecular picture. On the other hand, it could not be readily excluded for $D_s\bar{D}_s$ or $D_s^*\bar{D}_s$ as possible molecular configuration for $Y(4140)$ without explicit dynamics calculations. Catalyzed by the above reasons, we devote to calculate the spectra of the $(Q\bar{s})^{(*)}(\bar{Q}s)^{(*)}$ molecular states through QCD sum rules, to see whether $Y(4140)$ can be figured as a molecular state. In our approach, the masses for $D_s\bar{D}_s$, $D_s^*\bar{D}_s$, $D_s^*\bar{D}_s^*$, $B_s\bar{B}_s$, $B_s^*\bar{B}_s$, and $B_s^*\bar{B}_s^*$ molecular states are gained. In addition, to improve on the accuracy of QCD sum rule analysis for $Y(4140)$, the m_s^2 order and $\langle g^3G^3\rangle$ contributions are included in OPE side.

The paper is organized as follows. In Sec. II, QCD sum rules for the molecular states are introduced, and both the phenomenological representation and QCD side are derived, followed by the numerical analysis to extract the hadronic masses in Sec. III. Section IV is a brief summary.

II. $(Q\bar{s})^{(*)}(\bar{Q}s)^{(*)}$ QCD SUM RULES

The QCD sum rule attempts to link the hadron phenomenology with the interactions of quarks and gluons, which contains three main ingredients: an approximate description of the correlator in terms of intermediate states through the dispersion relation, a description of the same correlator in terms of QCD degrees of freedom via an OPE, and a procedure for matching these two descriptions and extracting the parameters that characterize the hadronic state of interest.

A. the molecular state QCD sum rule

In the molecular pictures, following forms of currents can be constructed for $(Q\bar{s})^{(*)}(\bar{Q}s)^{(*)}$ states, with

$$\begin{aligned} j_{(Q\bar{s})(\bar{Q}s)} &= (\bar{s}_a i\gamma_5 Q_a)(\bar{Q}_b i\gamma_5 s_b), \\ j_{(Q\bar{s})^*(\bar{Q}s)^*} &= (\bar{s}_a \gamma_\mu Q_a)(\bar{Q}_b \gamma^\mu s_b), \end{aligned}$$

for one type of hadrons, and

$$j^\mu_{(Q\bar{s})^*(\bar{Q}s)} = (\bar{s}_a \gamma^\mu Q_a)(\bar{Q}_b i\gamma_5 s_b),$$

for another type, where a and b are color indices.

For the former case, the starting point is the two-point correlator

$$\Pi(q^2) = i\int d^4x e^{iq.x}\langle 0|T[j(x)j^+(0)]|0\rangle. \tag{1}$$

The correlator can be phenomenologically expressed as a dispersion integral over a physical spectral function

$$\Pi(q^2) = \frac{\lambda_H^2}{M_H^2 - q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\Pi^{\mathrm{phen}}(s)}{s-q^2} + \text{subtractions}, \tag{2}$$

where M_H denotes the mass of the hadronic resonance, and λ_H gives the coupling of the current to the hadron $\langle 0|j|H\rangle = \lambda_H$. In the OPE side, the correlator can be written in terms of a dispersion relation as

$$\Pi(q^2) = \int_{(2m_Q+2m_s)^2}^{\infty} ds \frac{\rho^{\mathrm{OPE}}(s)}{s-q^2}, \tag{3}$$

where the spectral density is given by the imaginary part of the correlator

$$\rho^{\mathrm{OPE}}(s) = \frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{OPE}}(s). \tag{4}$$

After equating the two sides, assuming quark-hadron duality, and making a Borel transform, the sum rule can be written as

$$\lambda_H^2 e^{-M_H^2/M^2} = \int_{(2m_Q+2m_s)^2}^{s_0} ds \rho^{\mathrm{OPE}}(s) e^{-s/M^2}. \tag{5}$$

To eliminate the hadronic coupling constant λ_H, one reckons the ratio of derivative of the sum rule and itself, and then yields

$$M_H^2 = \int_{(2m_Q+2m_s)^2}^{s_0} ds \rho^{\mathrm{OPE}} s e^{-s/M^2} \Big/ \int_{(2m_Q+2m_s)^2}^{s_0} ds \rho^{\mathrm{OPE}} e^{-s/M^2}. \tag{6}$$

For the latter case, one starts from the two-point correlator

$$\Pi^{\mu\nu}(q^2) = i\int d^4x e^{iq.x}\langle 0|T[j^\mu(x)j^{\nu+}(0)]|0\rangle. \tag{7}$$

Lorentz covariance implies that the two-point correlation function can be generally parameterized as

$$\Pi^{\mu\nu}(q^2) = (\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu})\Pi^{(1)}(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi^{(0)}(q^2). \tag{8}$$

The part of the correlator proportional to $g_{\mu\nu}$ will be chosen to extract the mass sum rule here. In phenomenology, $\Pi^{(1)}(q^2)$ can be expressed as a dispersion integral over a physical spectral function

$$\Pi^{(1)}(q^2) = \frac{[\lambda^{(1)}]^2}{M_H^2 - q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\Pi^{(1)\mathrm{phen}}(s)}{s-q^2} + \text{subtractions}, \tag{9}$$

where M_H denotes the mass of the hadronic resonance. In the OPE side, $\Pi^{(1)}(q^2)$ can be written in terms of a dispersion relation as

$$\Pi^{(1)}(q^2)=\int_{(2m_Q+2m_s)^2}^{\infty}ds\frac{\rho^{\mathrm{OPE}}(s)}{s-q^2}, \tag{10}$$

where the spectral density is given by

$$\rho^{\mathrm{OPE}}(s)=\frac{1}{\pi}\mathrm{Im}\Pi^{(1)}(s). \tag{11}$$

After equating the two sides, assuming quark-hadron duality, and making a Borel transform, the sum rule can be written as

$$[\lambda^{(1)}]^2e^{-M_H^2/M^2} \;=\; \int_{(2m_Q+2m_s)^2}^{s_0}ds\rho^{\mathrm{OPE}}(s)e^{-s/M^2}. \tag{12}$$

To eliminate the hadronic coupling constant $\lambda^{(1)}$, one reckons the ratio of derivative of the sum rule and itself, and then yields

$$M_H^2 \;=\; \int_{(2m_Q+2m_s)^2}^{s_0}ds\rho^{\mathrm{OPE}}se^{-s/M^2}\Big/\int_{(2m_Q+2m_s)^2}^{s_0}ds\rho^{\mathrm{OPE}}e^{-s/M^2}. \tag{13}$$

B. spectral densities

Calculating the OPE side, one works at leading order in α_s and considers condensates up to dimension six with the similar techniques in Refs. [15, 16]. The s quark is dealt as a light one and the diagrams are considered up to order m_s^2. To keep the heavy-quark mass finite, one uses the momentum-space expression for the heavy-quark propagator. One calculates the light-quark part of the correlation function in the coordinate space, which is then Fourier-transformed to the momentum space in D dimension. The resulting light-quark part is combined with the heavy-quark part before it is dimensionally regularized at $D=4$. For the heavy-quark propagator with two and three gluons attached, the momentum-space expressions given in Ref. [17] are used. After some tedious calculations, finally with

$$\rho^{\mathrm{OPE}}(s)=\rho^{\mathrm{pert}}(s)+\rho^{\langle\bar{s}s\rangle}(s)+\rho^{\langle\bar{s}s\rangle^2}(s)+\rho^{\langle g\bar{s}\sigma\cdot Gs\rangle}(s)+\rho^{\langle g^2G^2\rangle}(s)+\rho^{\langle g^3G^3\rangle}(s),$$

$$\begin{aligned}\rho^{\mathrm{pert}}(s) \;=\;& \frac{3}{2^{11}\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^3}(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]^4\\ &-\frac{3}{2^8\pi^6}m_Qm_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^2}(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]^3\\ &+\frac{3^2}{2^9\pi^6}m_Q^2m_s^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^2}(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]^2,\end{aligned}$$

$$\begin{aligned}\rho^{\langle\bar{s}s\rangle}(s) \;=\;& -\frac{3\langle\bar{s}s\rangle}{2^6\pi^4}m_Q\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}[(\alpha+\beta)m_Q^2-\alpha\beta s]^2\\ &+\frac{3\langle\bar{s}s\rangle}{2^7\pi^4}m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha(1-\alpha)}[m_Q^2-\alpha(1-\alpha)s]^2\\ &+\frac{3\langle\bar{s}s\rangle}{2^5\pi^4}m_Q^2m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}[(\alpha+\beta)m_Q^2-\alpha\beta s]\\ &-\frac{3\langle\bar{s}s\rangle}{2^6\pi^4}m_Qm_s^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{1-\alpha}[m_Q^2-\alpha(1-\alpha)s],\end{aligned}$$

$$\begin{aligned}\rho^{\langle\bar{s}s\rangle^2}(s) &= \frac{\langle\bar{s}s\rangle^2}{2^4\pi^2}m_Q^2\sqrt{1-4m_Q^2/s} \\ &- \frac{\langle\bar{s}s\rangle^2}{2^4\pi^2}m_Q m_s\sqrt{1-4m_Q^2/s} \\ &+ \frac{3\langle\bar{s}s\rangle^2}{2^5\pi^2}m_s^2\int_{\alpha_{min}}^{\alpha_{max}} d\alpha\alpha(1-\alpha),\end{aligned}$$

$$\begin{aligned}\rho^{\langle g\bar{s}\sigma\cdot Gs\rangle}(s) &= -\frac{3\langle g\bar{s}\sigma\cdot Gs\rangle}{2^7\pi^4}m_Q\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{1-\alpha}[m_Q^2-\alpha(1-\alpha)s] \\ &+ \frac{\langle g\bar{s}\sigma\cdot Gs\rangle}{2^7\pi^4}m_s\int_{\alpha_{min}}^{\alpha_{max}} d\alpha[2m_Q^2-3\alpha(1-\alpha)s] \\ &+ \frac{3\langle g\bar{s}\sigma\cdot Gs\rangle}{2^7\pi^4}m_Q^2 m_s\sqrt{1-4m_Q^2/s} \\ &- \frac{\langle g\bar{s}\sigma\cdot Gs\rangle}{2^7\pi^4}m_Q m_s^2\sqrt{1-4m_Q^2/s},\end{aligned}$$

$$\begin{aligned}\rho^{\langle g^2G^2\rangle}(s) &= \frac{\langle g^2G^2\rangle}{2^{10}\pi^6}m_Q^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s] \\ &- \frac{3\langle g^2G^2\rangle}{2^{10}\pi^6}m_Q m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s] \\ &- \frac{\langle g^2G^2\rangle}{2^{10}\pi^6}m_Q^3 m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(\alpha+\beta)(1-\alpha-\beta) \\ &+ \frac{3\langle g^2G^2\rangle}{2^{10}\pi^6}m_Q^2 m_s^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta),\end{aligned}$$

$$\begin{aligned}\rho^{\langle g^3G^3\rangle}(s) &= \frac{\langle g^3G^3\rangle}{2^{12}\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s] \\ &+ \frac{\langle g^3G^3\rangle}{2^{11}\pi^6}m_Q^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta\beta(1-\alpha-\beta) \\ &- \frac{\langle g^3G^3\rangle}{2^{12}\pi^6}m_Q m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(\alpha+6\beta)(1-\alpha-\beta),\end{aligned}$$

for $(Q\bar{s})(\bar{Q}s)$,

$$\rho^{\rm OPE}(s) = -\{\rho^{\rm pert}(s)+\rho^{\langle\bar{s}s\rangle}(s)+\rho^{\langle\bar{s}s\rangle^2}(s)+\rho^{\langle g\bar{s}\sigma\cdot Gs\rangle}(s)+\rho^{\langle g^2G^2\rangle}(s)+\rho^{\langle g^3G^3\rangle}(s)\},$$

$$\begin{aligned}\rho^{\rm pert}(s) &= -\frac{3}{2^{12}\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^3}(1-\alpha-\beta)(1+\alpha+\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]^4 \\ &+ \frac{3}{2^{10}\pi^6}m_Q m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^2}(1-\alpha-\beta)(3+\alpha+\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]^3 \\ &- \frac{3^2}{2^9\pi^6}m_Q^2 m_s^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^2}(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]^2,\end{aligned}$$

$$\begin{aligned}
\rho^{\langle\bar{s}s\rangle}(s) &= \frac{3\langle\bar{s}s\rangle}{2^7\pi^4}m_Q\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}(1+\alpha+\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]^2 \\
&+ \frac{3\langle\bar{s}s\rangle}{2^7\pi^4}m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}[(\alpha+\beta)m_Q^2-\alpha\beta s]^2 \\
&- \frac{3\langle\bar{s}s\rangle}{2^7\pi^4}m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha(1-\alpha)}[m_Q^2-\alpha(1-\alpha)s]^2 \\
&- \frac{3\langle\bar{s}s\rangle}{2^5\pi^4}m_Q^2m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}[(\alpha+\beta)m_Q^2-\alpha\beta s] \\
&+ \frac{3\langle\bar{s}s\rangle}{2^6\pi^4}m_Qm_s^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{1-\alpha}[m_Q^2-\alpha(1-\alpha)s] \\
&- \frac{3\langle\bar{s}s\rangle}{2^7\pi^4}m_Qm_s^2\int_{\alpha_{min}}^{\alpha_{max}}d\alpha\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}[(\alpha+\beta)m_Q^2-\alpha\beta s],
\end{aligned}$$

$$\begin{aligned}
\rho^{\langle\bar{s}s\rangle^2}(s) &= -\frac{\langle\bar{s}s\rangle^2}{2^4\pi^2}m_Q^2\sqrt{1-4m_Q^2/s} \\
&+ \frac{3\langle\bar{s}s\rangle^2}{2^6\pi^2}m_Qm_s\sqrt{1-4m_Q^2/s} \\
&- \frac{3\langle\bar{s}s\rangle^2}{2^6\pi^2}m_s^2\int_{\alpha_{min}}^{\alpha_{max}}d\alpha\alpha(1-\alpha),
\end{aligned}$$

$$\begin{aligned}
\rho^{\langle g\bar{s}\sigma\cdot Gs\rangle}(s) &= -\frac{3\langle g\bar{s}\sigma\cdot Gs\rangle}{2^8\pi^4}m_Q\int_{\alpha_{min}}^{\alpha_{max}}d\alpha\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}[(\alpha+\beta)m_Q^2-\alpha\beta s] \\
&+ \frac{3\langle g\bar{s}\sigma\cdot Gs\rangle}{2^7\pi^4}m_Q\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{1-\alpha}[m_Q^2-\alpha(1-\alpha)s] \\
&- \frac{\langle g\bar{s}\sigma\cdot Gs\rangle}{2^7\pi^4}m_s\int_{\alpha_{min}}^{\alpha_{max}}d\alpha[m_Q^2-2\alpha(1-\alpha)s] \\
&- \frac{3\langle g\bar{s}\sigma\cdot Gs\rangle}{2^7\pi^4}m_Q^2m_s\sqrt{1-4m_Q^2/s} \\
&+ \frac{3\langle g\bar{s}\sigma\cdot Gs\rangle}{2^9\pi^4}m_Qm_s^2\sqrt{1-4m_Q^2/s},
\end{aligned}$$

$$\begin{aligned}
\rho^{\langle g^2G^2\rangle}(s) &= -\frac{\langle g^2G^2\rangle}{2^{11}\pi^6}m_Q^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)(1+\alpha+\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s] \\
&+ \frac{3\langle g^2G^2\rangle}{2^{12}\pi^6}m_Qm_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)(3+\alpha+\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s] \\
&+ \frac{\langle g^2G^2\rangle}{2^{12}\pi^6}m_Q^3m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(\alpha+\beta)(1-\alpha-\beta)(3+\alpha+\beta) \\
&- \frac{3\langle g^2G^2\rangle}{2^{10}\pi^6}m_Q^2m_s^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta),
\end{aligned}$$

$$\begin{aligned}
\rho^{\langle g^3G^3\rangle}(s) &= -\frac{\langle g^3G^3\rangle}{2^{13}\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)(1+\alpha+\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s] \\
&- \frac{\langle g^3G^3\rangle}{2^{12}\pi^6}m_Q^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta\beta(1-\alpha-\beta)(1+\alpha+\beta) \\
&+ \frac{\langle g^3G^3\rangle}{2^{14}\pi^6}m_Qm_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(\alpha+6\beta)(1-\alpha-\beta)(3+\alpha+\beta),
\end{aligned}$$

for $(Q\bar{s})^*(\bar{Q}s)$, and

$$\rho^{\rm OPE}(s)=\rho^{\rm pert}(s)+\rho^{\langle\bar{s}s\rangle}(s)+\rho^{\langle\bar{s}s\rangle^2}(s)+\rho^{\langle g\bar{s}\sigma\cdot Gs\rangle}(s)+\rho^{\langle g^2G^2\rangle}(s)+\rho^{\langle g^3G^3\rangle}(s),$$

$$\begin{aligned}\rho^{\rm pert}(s) &= \frac{3}{2^9\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^3}(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]^4\\ &-\frac{3}{2^7\pi^6}m_Qm_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^2}(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]^3\\ &+\frac{3^2}{2^7\pi^6}m_Q^2m_s^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^2}(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]^2,\end{aligned}$$

$$\begin{aligned}\rho^{\langle\bar{s}s\rangle}(s) &= -\frac{3\langle\bar{s}s\rangle}{2^5\pi^4}m_Q\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}[(\alpha+\beta)m_Q^2-\alpha\beta s]^2\\ &+\frac{3\langle\bar{s}s\rangle}{2^5\pi^4}m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha(1-\alpha)}[m_Q^2-\alpha(1-\alpha)s]^2\\ &+\frac{3\langle\bar{s}s\rangle}{2^3\pi^4}m_Q^2m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}[(\alpha+\beta)m_Q^2-\alpha\beta s]\\ &-\frac{3\langle\bar{s}s\rangle}{2^5\pi^4}m_Qm_s^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{1-\alpha}[m_Q^2-\alpha(1-\alpha)s],\end{aligned}$$

$$\begin{aligned}\rho^{\langle\bar{s}s\rangle^2}(s) &= \frac{\langle\bar{s}s\rangle^2}{2^2\pi^2}m_Q^2\sqrt{1-4m_Q^2/s}\\ &-\frac{\langle\bar{s}s\rangle^2}{2^3\pi^2}m_Qm_s\sqrt{1-4m_Q^2/s}\\ &+\frac{3\langle\bar{s}s\rangle^2}{2^3\pi^2}m_s^2\int_{\alpha_{min}}^{\alpha_{max}}d\alpha\alpha(1-\alpha),\end{aligned}$$

$$\begin{aligned}\rho^{\langle g\bar{s}\sigma\cdot Gs\rangle}(s) &= -\frac{3\langle g\bar{s}\sigma\cdot Gs\rangle}{2^6\pi^4}m_Q\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{1-\alpha}[m_Q^2-\alpha(1-\alpha)s]\\ &+\frac{\langle g\bar{s}\sigma\cdot Gs\rangle}{2^5\pi^4}m_s\int_{\alpha_{min}}^{\alpha_{max}}d\alpha[2m_Q^2-3\alpha(1-\alpha)s]\\ &+\frac{3\langle g\bar{s}\sigma\cdot Gs\rangle}{2^5\pi^4}m_Q^2m_s\sqrt{1-4m_Q^2/s}\\ &-\frac{\langle g\bar{s}\sigma\cdot Gs\rangle}{2^6\pi^4}m_Qm_s^2\sqrt{1-4m_Q^2/s},\end{aligned}$$

$$\begin{aligned}\rho^{\langle g^2G^2\rangle}(s) &= \frac{\langle g^2G^2\rangle}{2^8\pi^6}m_Q^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]\\ &-\frac{3\langle g^2G^2\rangle}{2^9\pi^6}m_Qm_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]\\ &-\frac{\langle g^2G^2\rangle}{2^9\pi^6}m_Q^3m_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(\alpha+\beta)(1-\alpha-\beta)\\ &+\frac{3\langle g^2G^2\rangle}{2^8\pi^6}m_Q^2m_s^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta),\end{aligned}$$

$$\rho^{\langle g^3G^3\rangle}(s) = \frac{\langle g^3G^3\rangle}{2^{10}\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)[(\alpha+\beta)m_Q^2-\alpha\beta s]$$
$$+\frac{\langle g^3G^3\rangle}{2^9\pi^6}m_Q^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta\beta(1-\alpha-\beta)$$
$$-\frac{\langle g^3G^3\rangle}{2^{11}\pi^6}m_Qm_s\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(\alpha+6\beta)(1-\alpha-\beta),$$

for $(Q\bar{s})^*(\bar{Q}s)^*$. The integration limits are given by $\alpha_{min} = (1-\sqrt{1-4m_Q^2/s})/2$, $\alpha_{max} = (1+\sqrt{1-4m_Q^2/s})/2$, and $\beta_{min} = \alpha m_Q^2/(s\alpha - m_Q^2)$.

III. NUMERICAL ANALYSIS

In this part, the sum rules (6) and (13) will be numerically analyzed. The input values are taken as $m_c = 1.23$ GeV, $m_b = 4.20$ GeV, $m_s = 0.13$ GeV, $\langle\bar{q}q\rangle = -(0.23)^3$ GeV^3, $\langle\bar{s}s\rangle = 0.8\ \langle\bar{q}q\rangle$, $\langle g\bar{s}\sigma\cdot Gs\rangle = m_0^2\ \langle\bar{s}s\rangle$, $m_0^2 = 0.8$ GeV^2, $\langle g^2G^2\rangle = 0.88$ GeV^4, and $\langle g^3G^3\rangle = 0.045$ GeV^6. Complying with the standard procedure of sum rule analysis, the threshold s_0 and Borel parameter M^2 are varied to find the optimal stability window, in which the perturbative contribution should be larger than the condensate contributions while the pole contribution larger than continuum contribution. Thus, the regions of s_0 and M^2 are taken as $\sqrt{s_0} = 4.3 \sim 4.5$ GeV, $M^2 = 3.5 \sim 4.5$ GeV^2 for $D_s\bar{D}_s$, $\sqrt{s_0} = 4.5 \sim 4.7$ GeV, $M^2 = 3.5 \sim 4.5$ GeV^2 for $D_s^*\bar{D}_s$, $\sqrt{s_0} = 4.6 \sim 4.8$ GeV, $M^2 = 3.5 \sim 4.5$ GeV^2 for $D_s^*\bar{D}_s^*$, $\sqrt{s_0} = 11.1 \sim 11.3$ GeV, $M^2 = 9.5 \sim 11.0$ GeV^2 for $B_s\bar{B}_s$, $\sqrt{s_0} = 11.1 \sim 11.3$ GeV, $M^2 = 9.5 \sim 11.0$ GeV^2 for $B_s^*\bar{B}_s$, and $\sqrt{s_0} = 11.2 \sim 11.4$ GeV, $M^2 = 9.5 \sim 11.0$ GeV^2 for $B_s^*\bar{B}_s^*$, respectively. The corresponding Borel curves are exhibited in Figs. 1-3. Ultimately, we obtain the mass values: 3.91 ± 0.10 GeV for $D_s\bar{D}_s$, 4.01 ± 0.10 GeV for $D_s^*\bar{D}_s$, 4.13 ± 0.10 GeV for $D_s^*\bar{D}_s^*$, 10.70 ± 0.10 GeV for $B_s\bar{B}_s$, 10.71 ± 0.11 GeV for $B_s^*\bar{B}_s$, and 10.80 ± 0.10 GeV for $B_s^*\bar{B}_s^*$. It is worth noting that uncertainty in our results are merely owing to the sum rule windows (variation of the threshold s_0 and Borel parameter M^2), not involving the ones from the variation of quark masses and QCD parameters.

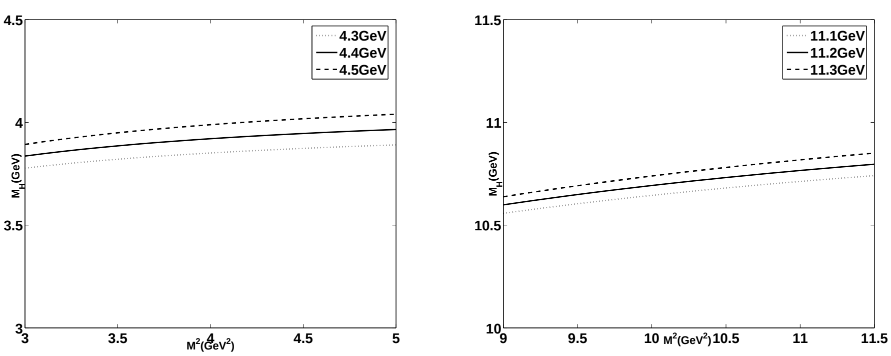

FIG. 1: The dependence on M^2 for the masses of $D_s\bar{D}_s$ and $B_s\bar{B}_s$ from sum rule (6). The continuum thresholds are taken as $\sqrt{s_0} = 4.3 \sim 4.5$ GeV and $\sqrt{s_0} = 11.1 \sim 11.3$ GeV, respectively.

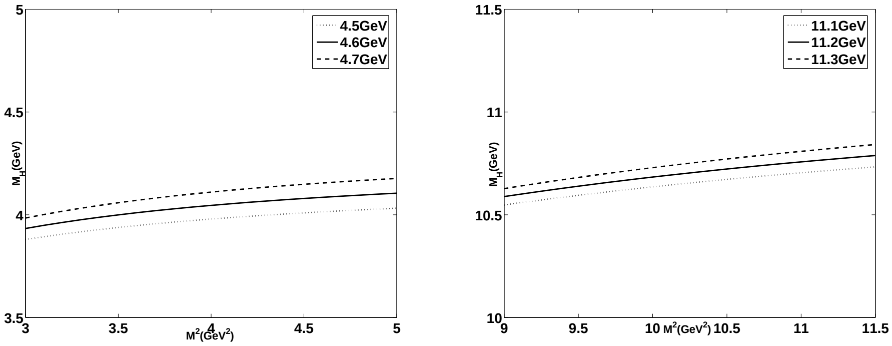

FIG. 2: The dependence on M^2 for the masses of $D_s^*\bar{D}_s$ and $B_s^*\bar{B}_s$ from sum rule (13). The continuum thresholds are taken as $\sqrt{s_0} = 4.5 \sim 4.7$ GeV and $\sqrt{s_0} = 11.1 \sim 11.3$ GeV, respectively.

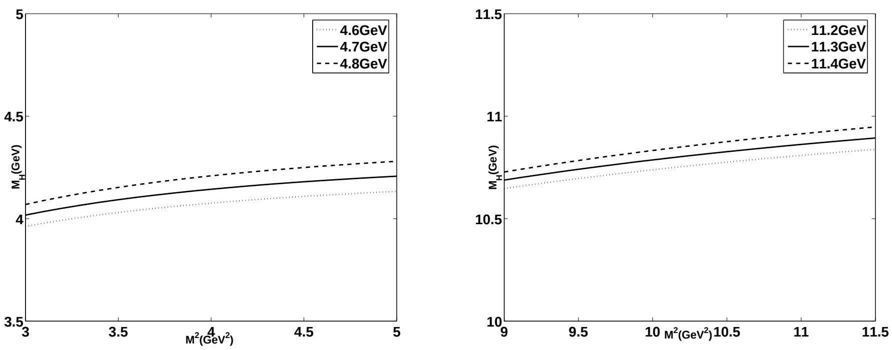

FIG. 3: The dependence on M^2 for the masses of $D_s^*\bar{D}_s^*$ and $B_s^*\bar{B}_s^*$ from sum rule (6). The continuum thresholds are taken as $\sqrt{s_0} = 4.6 \sim 4.8$ GeV and $\sqrt{s_0} = 11.2 \sim 11.4$ GeV, respectively.

IV. SUMMARY

In summary, the QCD sum rules have been employed to compute the masses of $(Q\bar{s})^{(*)}(\bar{Q}s)^{(*)}$, including the contributions of the operators up to dimension six in OPE. For the charmed molecular states, we have got $M_{D_s\bar{D}_s} = 3.91 \pm 0.10$ GeV, $M_{D_s^*\bar{D}_s} = 4.01 \pm 0.10$ GeV, and $M_{D_s^*\bar{D}_s^*} = 4.13 \pm 0.10$ GeV. The numerical values for $D_s\bar{D}_s$ and $D_s^*\bar{D}_s$ are lower than the mass of $Y(4140)$, $4143.0 \pm 2.9 \pm 1.2$ MeV, whereas, the one for $D_s^*\bar{D}_s^*$ is well compatible with the experimental data, which supports the $D_s^*\bar{D}_s^*$ configuration for $Y(4140)$. Additionally, we have extracted $M_{B_s\bar{B}_s} = 10.70 \pm 0.10$ GeV, $M_{B_s^*\bar{B}_s} = 10.71 \pm 0.11$ GeV, and $M_{B_s^*\bar{B}_s^*} = 10.80 \pm 0.10$ GeV for the bottom molecular states. Altogether, all these theoretical results are looking forward to further experimental identification.

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